High Energy Physics

Lecture 4
More kinematics and a picture show of particle collisions
Recall from the previous lecture: the momentum of the scattered Particle in an elastic collision is given by

\[
p_c = p \frac{(s + m_a^2 - m_b^2) \cos \theta \pm 2W \sqrt{m_b^2 - m_a^2 \sin^2 \theta}}{2(s + p^2 \sin^2 \theta)}
\]

Note that \(p_c\) must be real and non-negative. Reality of \(p_c\) requires the radicand to be non-negative, and this means that

\[
\sin \theta \leq \frac{m_b}{m_a}
\]

Now if \(m_b/m_a > 1\), then all angles \(0 < \theta < 180\) degrees are acceptable, but the lower sign in front of the square root must be rejected as this could give unphysical negative values of \(p_c\).
Polar diagram of the LAB kinematics of elastic $\pi p$ scattering: $p_c$ is the momentum of the scattered pion; the scattering angle $\theta$ can take on all values from 0 to 180 degrees; only the upper sign in front of the square root in the formula for $p_c$ is acceptable; the lower sign gives negative values of $p_c$ and must be rejected.

$E_{LAB} = 1000 \text{ MeV}$; $m_p = 940 \text{ MeV}$, $m_\pi = 140 \text{ MeV}$

Note the different scales of the horizontal and vertical axes!
Polar diagram of the LAB kinematics of elastic $\alpha p$ scattering:
$p_c$ is the momentum of the scattered $\alpha$ particle; the scattering angle $\theta$ can take on all values from 0 to $\theta_{\text{max}}$; both signs in front of the square root in the formula for $p_c$ are acceptable: they both give positive values of $p_c$.
$E_{\text{LAB}} = 10$ GeV; $m_p = 0.94$ GeV, $m_\alpha = 3.76$ GeV; $\theta_{\text{max}} = 15.5$ deg.
Note the different scales along the horizontal and vertical axes!
Kinematics of particle decay

We need the kinematics of particle decays to understand such pictures of particle reactions as are shown in the following examples of bubble chamber pictures where we will see the tracks created by the passage of ionizing particles in liquid hydrogen and in freon.

The bubble chamber volume is filled with a strong magnetic field, so the charged particle tracks are curved.

From the curvature one can reconstruct the particle momentum. From the density of ionization along the tracks one can also find the energy of the particle. (We will discuss this in another lecture.) Knowing energy and momentum of the particle one can deduce the particle mass using the relativistic energy-momentum relation

\[ E^2 - p^2 = m^2 \]

Neutral particles leave no tracks; their energies, momenta and masses can be found only by applying energy and momentum conservation to their decay products.

This is a complicated task whenever the decay products are themselves neutral particles!
Two-body decay of unstable particle:

Mother particle, mass = \( M \)

Daughter particles, masses = \( m_1, m_2 \)

4-momenta: \( P=(M,0,0,0), \ p_1=(E_1,p_{1x},p_{1y},p_{1z}), \ p_2=(E_2,p_{2x},p_{2y},p_{2z}), \)

4-momentum conservation (or energy-momentum conservation):

\[
P = p_1 + p_2
\]

Momentum conservation:

\[
\bar{p}_1 + \bar{p}_2 = 0 \quad \text{hence} \quad p = |\bar{p}_1| = |\bar{p}_2|
\]

Energy conservation:

\[
M = E_1 + E_2 = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2}
\]
\[ p = \frac{1}{2M} \sqrt{\left[ M^2 - (m_1 - m_2)^2 \right] \left[ M^2 - (m_1 + m_2)^2 \right]} \]

Reality of \( p \) implies: \( M > m_1 + m_2 \)

for the decay to take place.

But it must be understood: if a decay is possible kinematically but is not observed, then it must be forbidden by a conservation law.

**Two-body decay in flight:**

for example a \( B \) meson decaying in flight into a pair of pions

\[ P = (E, 0, 0, p), \quad p_1 = (E_1, p_{1x}, p_{1y}, p_{1z}), \quad p_2 = (E_2, p_{2x}, p_{2y}, p_{2z}), \]

Lorentz boost from CMS to LAB:

\[ E_1 = \gamma \left( E_{1}^* + \nu p_{1z}^* \right), \]

\[ p_{1z} = \gamma \left( p_{1z}^* + \nu E_{1}^* \right) \]

\[ \vec{p}_{1T} = \vec{p}_{1T}^* \]
\[ v = p / E, \quad \gamma = E / M \]

hence can find magnitude of momentum of daughter particle 1:

\[
p_1 = \frac{1}{2} \frac{(M^2 + m_1^2 - m_2^2)p \cos \theta_1 \pm 2E \sqrt{M^2 p^2 - m_1^2 p^2 \sin^2 \theta_1}}{M^2 + p^2 \sin^2 \theta_1}
\]

Where \( \theta_1 \) is the direction of daughter 1 with the line of flight of the mother particle.

(Exercise!)

similar formula for daughter 2

Such formulae are needed to interpret the bubble chamber pictures in the following show. But first ...
... what’s a bubble chamber?

A bubble chamber is a vessel filled with a superheated liquid. Even a very small disturbance can cause boiling of the liquid.

Such a disturbance can be the ionization along the path of a charged particle: boiling will begin along that path forming a visible track of bubbles that can be photographed.

The photograph must be taken before boiling spreads over the entire volume of the liquid.

Once the entire volume of liquid boils, the chamber must be prepared for the next passage of particles through it. This is done by raising the pressure and hence the boiling point, then carefully expanding to lower the boiling point below the temperature of the liquid.

The entire cycle between two states of readiness of the chamber takes several seconds; so the bubble chamber is a fairly slow tool: if you are lucky you can take of the order of one million bubble chamber pictures in one year.
The biggest ever bubble chamber was BEBC: the Big European Bubble Chamber.

Its working volume was 20 cubic meters.

It was surrounded by a superconducting solenoid producing a magnetic field of 3.5 Tesla.

The working liquid was hydrogen whose boiling point is 20.26 K.

The solenoid was surrounded by an array of muon chambers.

BEBC was exposed to an intensive neutrino beam.

BEBC was decommissioned in 1984.
one of the $\gamma$s converts into an $e^+e^-$ pair; the neutron and the photons leave no trails; due to the short lifetime of the neutral pion, the reconstructed photon momentum points straight at the interaction vertex. Actual picture on the left, schematic on the right. Additional tracks must be ignored.
Note: In order to get a 3D view of the event, one takes pictures with two or three cameras under different angles; then one can identify spurious tracks, not associated with the “interesting” event, and measure the curvatures of the tracks (helixes!).
Bubble chamber picture of a Dalitz decay of a neutral pion

Fig. 1.7 Example of the Dalitz decay of a neutral pion, $\pi^0 \rightarrow \gamma + e^+e^-$, in a bubble chamber. The $\pi^0$ is created at the point $A$, in a $\pi^-p$ inelastic collision. The $\pi^0$ travels a distance of <1 $\mu$ in a mean lifetime, so that the $e^+e^-$ pair appears to come directly from the interaction.

Note: neutral pions usually decay into two real photons but about 1% of pi-zeros decay into one real and one virtual photon which instantly breaks up into an electron-positron pair ("Dalitz decay", called after Richard Dalitz).

In the bubble chamber picture the second - real - photon from the pi-zero decay did not get converted within the bubble chamber volume ("fiducial volume") and hence escaped detection.

The next picture is of the famous event of the discovery of the $\Omega^-$ baryon:
The first $\Omega^-$ event (Barnes et al. 1964)

Barnes et al., Phys. Rev. Letters 12 (1964) 204

The \( \Omega^- \) baryon (of strangeness \( S = -3 \)) was theoretically predicted by Murray Gell-Mann on the grounds of his SU(3) model of particle classification. His prediction included the approximate mass of the \( \Omega^- \), its decay modes and a rough idea of its lifetime.

The \( \Omega^- \) is produced in a strong interaction that conserves strangeness:

\[
K^- p \rightarrow \Omega^- K^+ K^0
\]

\[
S : \quad -1 + 0 = S_{\Omega} + 1 + 1
\]

hence

\[
S_{\Omega} = -3
\]

But the \( \Omega^- \) cannot decay strongly; indeed, try the decay into a baryon of strangeness \( -2 \) and an \( S = -1 \) meson:
$$\Omega^- \rightarrow \Xi^- + \bar{K}^0$$

This decay is forbidden by energy conservation:

$$m(\Omega^-) = 1672.45 \text{ MeV}, \quad m(\Xi^-) = 1321.31 \text{ MeV}, \quad m(\bar{K}^0) = 498 \text{ MeV}$$

hence

$$m(\Omega^-) < m(\Xi^-) + m(\bar{K}^0)$$

**Recall:** \(K^- = \bar{u}s; \quad K^+ = u\bar{s}; \quad K^0 = d\bar{s}; \quad \bar{K}^0 = \bar{d}s\)

The \(s\) quark has strangeness -1, therefore the anti-\(s\) quark has strangeness +1, and hence the \(K^-\) has strangeness -1 and the \(K^+\) and \(K^0\) have strangeness +1.

But weak decays are allowed since they proceed with a change of strangeness, e.g.

$$\Omega^- \rightarrow \Xi^0 + \pi^- \quad (\Delta S = 1)$$
Discovery of Neutral Currents (1973)

The next example is a picture of a neutral current event taken in the CERN heavy liquid bubble chamber Gargamelle, which was exposed to a muon-neutrino beam.

This and similar events confirmed in 1973 the existence of the neutral intermediate vector boson, predicted in the unified electroweak interaction theory of Glashow, Salam and Weinberg.

The muon neutrinos are incident from the left. The charged particle tracks are identified by their curvature (giving the momentum) and density of ionization (giving the energy). They look in the picture like being created “out of nothing”; no outgoing muon was seen in this event, so the outgoing lepton must be a muon neutrino to conserve the muon lepton number.
Neutral current event

Feynman diagram of a neutral current reaction;
X is any system of hadrons

(from Martin and Shaw, *Particle Physics*, 2nd edition)
Charged current event in the CERN heavy liquid bubble chamber

The neutrino comes in from below; it interacts with a nuclear neutron. The reaction is

\[ \nu_\mu n \rightarrow p \mu^- \]

Production of a charmed D* meson seen in the BEBC bubble chamber exposed to a neutrino beam at CERN.
The interpretation of an event like this does obviously present a serious problem of kinematics. Indeed, all that is seen by the experimentalist is the reaction

\[ \nu_\mu p \rightarrow \mu^- p \pi^+ \pi^+ K^- \]

(that the initial state hadron was a proton and not a neutron follows from charge balance).

The reaction is a CC weak interaction process; this follows from having a neutrino beam and a muon in the final state coming from the interaction vertex.

After that observation one must test any number of hypotheses, especially if one of the intermediate particles has not been seen previously. I hope that before the end of this course, if we come back to this event, we shall be able to understand it more deeply, including the dynamics of the interaction.