

Spontaneous breaking of chiral symmetry

Scales in strong interactions

'Strong interactions' are about everything from π 's to ^{238}U and to the interior of the compact stellar objects. Masses are measured in MeV:

$$m_e = 0.511 \text{ MeV}, \quad m_\pi = 140 \text{ MeV}, \quad m_N = 940 \text{ MeV}$$

Distances and sizes are measured in fm = 10^{-15} m: $r_\pi \approx r_N = 1$ fm.

Quantum Chromodynamics (QCD)

The microscopic theory of strong interactions is encoded in a one-line QCD Lagrangian [Fritzsch, Gell-Mann and Leutwyler (1972)]:

$$\mathcal{L} = \sum_{f=1}^6 \bar{q}_f (i\gamma_\mu \nabla_\mu - m_f) q_f - \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a,$$

$$\nabla_{\mu} = \frac{\partial}{\partial x_{\mu}} - iA_{\mu}^a t^a \quad a = 1, 2 \dots 8,$$

$$F_{\mu\nu}^a = \partial_{\mu}A_{\nu}^a - \partial_{\nu}A_{\mu}^a + f^{abc} A_{\mu}^b A_{\nu}^c.$$

q_f are quark fields, u, d, s, \dots , A_{μ}^a is the gluon field, $F_{\mu\nu}^a$ is the gluon field strength.

$m_u \approx 4 \text{ MeV}$, $m_d \approx 7 \text{ MeV}$, $m_s \approx 150 \text{ MeV}$, $m_c \approx 1250 \text{ MeV} \dots$

$$\left. \begin{array}{l} \text{proton } p \approx uud \\ \text{neutron } n \approx udd \end{array} \right\} m_N = 940 \text{ MeV}$$

How come the nucleon is 100 times heavier its constituents?

'Transmutation of dimensions'

$\alpha_s = \frac{g^2}{4\pi}$ is the analog of $\alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}$ but it 'runs':

$$\frac{2\pi}{\alpha_s(\mu)} = b_1 \ln \frac{\mu}{\Lambda} + \frac{b_2}{2b_1} \ln \ln \frac{\mu^2}{\Lambda^2} + O\left(\frac{1}{\ln \frac{\mu}{\Lambda}}\right).$$

$$b_1 = \frac{11}{3}N_c - \frac{2}{3}N_f, \quad b_2 = \frac{34}{3}N_c^2 - \frac{13}{3}N_cN_f + \frac{N_f}{N_c}.$$

All dimensionfull observables come out as combinations of the UV cutoff μ and the coupling constant given at that cutoff, $g^2(\mu)$.

$$\Lambda = \mu \exp\left(-\frac{2\pi}{b_1\alpha_s(\mu)}\right) \left(\frac{4\pi}{b\alpha_s(\mu)}\right)^{\frac{b_2}{2b_1^2}} (1 + O(\alpha_s(\mu))).$$

For example, $\mu = \frac{1}{a}$ lattice cutoff.

Chiral limit

– is an idealization: $m_u = m_d = m_s = 0$. Only 5% of the nucleon mass is due to $m \neq 0$. In this (idealized) world interaction of quarks with gluons conserves quark helicity or chirality: left-polarized u, d, s quarks remain left-polarized forever. In addition, u, d, s quarks are interchangeable.

Mathematically, the QCD lagrangian is invariant under $U(3)_L \times U(3)_R$ separate rotations of left- and right-polarized quarks,

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix}_L \rightarrow \begin{pmatrix} \times & \times & \times \\ \times & A & \times \\ \times & \times & \times \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}_L ,$$

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix}_R \rightarrow \begin{pmatrix} \times & \times & \times \\ \times & B & \times \\ \times & \times & \times \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}_R$$

A and B are 3×3 unitary matrices, $9+9=18$ parameters!

The invariance under chiral rotations is called the **chiral symmetry** of strong interactions.

Since $L \leftrightarrow R$ in the mirror image,

**CHIRAL SYMMETRY OF QCD means
ALL STATES WITH OPPOSITE PARITY
HAVE EQUAL MASSES**

In reality

ρ	meson	$(J^P = 1^-)$	$m = 770 \text{ MeV}$
a_1	meson	$(J^P = 1^+)$	$m = 1250 \text{ MeV}$
			difference \approx <u>500 MeV</u>

For the nucleons the splitting is even larger,

N	$(\frac{1}{2}^+)$	$m = 940 \text{ MeV}$
N^*	$(\frac{1}{2}^-)$	$m = 1535 \text{ MeV}$
		difference \approx <u>600 MeV</u>

The difference is too large to be explained by nonzero current quark masses ($m_u = 4 \text{ MeV}$, $m_d = 7 \text{ MeV}$).

⇒ chiral symmetry is **spontaneously** broken

⇒ pions are light [(pseudo)-Goldstone bosons]

⇒ nucleons are heavy.

⇒ nuclei exist

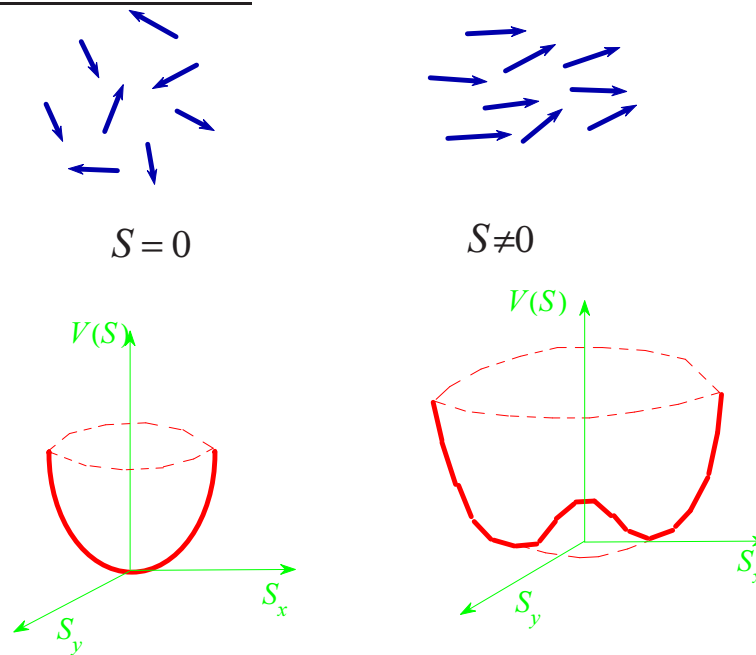
⇒ ... and we exist.

Were chiral symmetry unbroken, it would be the other way: pions would be heavy while nucleons would be light.

95% of the (visible) mass around us is due to the spontaneous chiral symmetry breaking.
It is a basic event in QCD.

Spontaneous breaking of any continuous symmetry is marked by a nonzero **order parameter**, in this case called **quark condensate** $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{s}s \rangle \neq 0$.

Analogy: Spontaneous magnetization



Order parameter $\langle S_x \rangle \neq 0$.

Symmetry with respect to rotation,

$$\begin{aligned} S_x &\rightarrow S_x \cos \phi + S_y \sin \phi, \\ S_y &\rightarrow -S_x \sin \phi + S_y \cos \phi \end{aligned}$$

is broken! As a result, there is one Goldstone or massless or gapless excitation in the system. The energy is $\mathcal{E} = \frac{1}{2}(\nabla\phi)^2$.

If magnetic field \mathbf{B} is added, rotational symmetry is broken from the start. The “Mexican hat” is tilted, and the energy gets a term

$$\begin{aligned} \mathcal{E}_{\text{pot}} &= -\mu(\mathbf{B} \cdot \mathbf{S}) = -\mu|B| \langle S \rangle \cos \phi \\ &= \mu|B| \langle S \rangle \left(-1 + \frac{1}{2}\phi^2\right) \end{aligned}$$

such that the Goldstone excitation gets a mass

$$m_G^2 = \mu|B| \langle S \rangle$$

Notice that m_G is **square root** of $|B|$.

With chiral symmetry, out of 18 symmetries 9 are spontaneously broken. It means that there must be **nine (pseudo) massless Goldstone bosons**. These are the pseudoscalar mesons:

$$\pi^+ \sim \bar{d}u, \quad \pi^- \sim \bar{u}d, \quad \pi^0 \sim \frac{\bar{u}u - \bar{d}d}{\sqrt{2}}$$

$$K^+ \sim \bar{s}u, \quad K^0 \sim \bar{s}d, \quad K^-, \quad \bar{K}^0$$

$$\eta \sim \frac{\bar{u}u + \bar{d}d - 2\bar{s}s}{\sqrt{6}}$$

$$\eta' \sim \frac{\bar{u}u + \bar{d}d + \bar{s}s}{\sqrt{3}}$$

Recall nonzero $m_{u,d,s}$ – analogs of magnetic field \vec{B} which breaks symmetry explicitly. It lifts the degeneracy of the bottom of the valley, and **makes Goldstone bosons massive**,

although still light (Gell-Mann–Oakes–Renner formula):

$$m_\pi^2 = \frac{1}{F^2}(m_u + m_d) \langle \bar{q}q \rangle = (140 \text{ MeV})^2$$

$$m_K^2 = \frac{1}{F^2}(m_s + m_{u(d)}) \langle \bar{q}q \rangle = (495 \text{ MeV})^2$$

$$\Rightarrow \frac{m_s}{m_u + m_d} \approx 12.5$$

$$\Rightarrow m_s \simeq 140 \text{ MeV}, \quad m_d \simeq 7 \text{ MeV}, \quad m_u \simeq 4 \text{ MeV}$$

A check:

$$\eta \sim \frac{\bar{u}u + \bar{d}d - 2\bar{s}s}{\sqrt{6}} \Rightarrow$$

$$m_\eta^2 \simeq \frac{4}{3} \frac{1}{F^2} m_s \langle \bar{q}q \rangle \simeq \frac{4}{3} m_K^2$$

547 MeV vs. 570 MeV, works well!

$$\eta' \sim \frac{\bar{u}u + \bar{d}d + \bar{s}s}{\sqrt{3}} \Rightarrow \quad (1)$$

$$m_{\eta'}^2 \simeq \frac{2}{3}m_K^2$$

958 MeV vs. 404 MeV, completely wrong!

This paradox was called “the $U(1)$ problem” [Weinberg (1974)!]

The mechanism of chiral symmetry breaking must be such that η' is NOT A GOLDSTONE BOSON.